A GENERALIZED ANISOTROPIC QUADRIC YIELD CRITERION AND ITS APPLICATION TO BONE AT MULTIPLE LENGTH SCALES

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Introduction

Nonlinear computational analysis of materials showing inelastic behaviour relies on knowledge of their yield behaviour and strength under complex stress states. One obstacle when dealing with elasto-plasticity is the choice of the correct yield surface for the given material. Therefore, it is desirable to use a formulation that is able to describe a wide range of materials while retaining a simple and explicit form. A possible class of surfaces is the set of convex quadrics. Bone tissue changes the shape of its yield envelope depending on the length scale considered. The aim of this study was to find a general formulation able to describe bone on several length scales.

Methods

A generalized anisotropic quadric yield criterion is proposed that is homogeneous of degree one and takes a convex quadric shape:

$$Y(\boldsymbol{S}) := \sqrt{\boldsymbol{S} : \mathbb{F}\boldsymbol{S}} + \boldsymbol{F} : \boldsymbol{S} - 1 = 0 \qquad (1)$$

For fabric- and density-based orthotropy, the tensors defining the criterion take the form:

$$\mathbb{F} = \sum_{i=1}^{3} F_i^2 \boldsymbol{M}_i \otimes \boldsymbol{M}_i - \sum_{i,j=1;i\neq j}^{3} \zeta_{ij} F_i^2 \boldsymbol{M}_i \otimes \boldsymbol{M}_j + \sum_{i,j=1;i\neq j}^{3} \frac{1}{2\tau_{ij}^2} \boldsymbol{M}_i \overline{\underline{\otimes}} \boldsymbol{M}_j \qquad (2)$$

$$\boldsymbol{F} = \sum_{i=1}^{3} \frac{1}{2} \left(\frac{1}{\sigma_{ii}^+} - \frac{1}{\sigma_{ii}^-} \right) \boldsymbol{M}_i. \qquad (3)$$

with

$$F_i = \frac{1}{2} \left(\frac{1}{\sigma_{ii}^+} + \frac{1}{\sigma_{ii}^-} \right) \qquad (4)$$

$$\sigma_{ii}^+ = \sigma_0^+ \rho^p m_i^{2q} \qquad \sigma_{ii}^- = \sigma_0^- \rho^p m_i^{2q} \qquad (5)$$

$$\zeta_{ij} = \zeta_0 \frac{m_i^{2q}}{m_j^{2q}} \qquad \tau_{ij} = \tau_0 \rho^p m_i^q m_j^q \qquad (6)$$

A fabric- and density-based quadric criterion for the description of homogenized material behaviour of trabecular bone was identified from uniaxial, multiaxial and torsional experimental data [Rincon 2009]. Finally, a quadric yield criterion for lamellar bone was identified based on a nanoindentation study from the literature [Carnelli 2011].

Results

A subset of the different possible shapes of the criterion in normal stress space is shown in Figure 1.

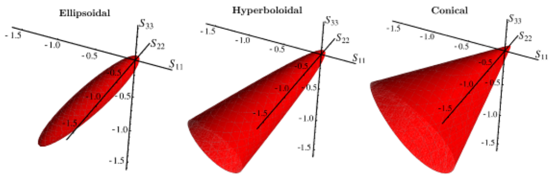

Figure 1: Subset of the possible shapes of the proposed quadric formulation.

The parameters defining a strength criterion for homogenized trabecular bone and a yield criterion for lamellar bone on the microscale are summarized in Table 1 and 2, respectively.

σ_0^{u+}	σ_0^{u-}	ζ_0^u	τ_0^u	p^u	q^u	N	R^2
39.7	53.2	0.226	22.9	1.29	0.593	95	0.933

Table 1: Parameters defining a quadric strength criterion for homogenized trabecular bone.

σ_0^+	σ_0^-	ζ_0	τ_0	p	q	m1=m2	m3
100	136	0.5	116.6	0.0	1.0	0.975	1.05

Table 2: Parameters defining a quadric yield criterion for lamellar bone on the microscale.

Discussion

The generality of the formulation is beneficial for material identification. If the shape of the yield function is not known a priori, a minimization using the proposed criterion will result in an optimal shape. The quadric yield criterion with its ability to take different convex quadric shapes is very suitable to approximate the yield envelope of bone at several hierarchical levels starting from the extracellular matrix on the microscale to the homogenized trabecular bone at the macroscale.

References

Rincon *et al*, Biomech Model Mechanobiol, 8:195-208, 2009.
Carnelli *et al*, J Biomech, 44:1852-58, 2011.