# A CRACK PHASE-FIELD APPROACH TO MODEL AORTIC DISSECTIONS

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## Introduction

An aortic dissection describes a loss of structural integrity in the artery wall. The initial phase of this cardiovascular disease is a formation of a tear in the intimal sub-layer. During a cardiac cycle the heart jets additional blood in this region, which leads to a separation of sub-layers, mainly in the media. The cause for the formation of a crack and its propagation during a physiological cardiac cycle is still unclear. The aim of this abstract is to demonstrate a numerical method for the simulation of a crack formation between two medial sub-layers, as observed in patients.

## Methods

The thermodynamic approach for the crack phase-field of fracture, as developed in [1] for an isotropic material, was used as a basis for the crack propagation model. The crack phase-field is described by the primary field variable $\boldsymbol{d}$, which is interpolated between the intact state of the material ($\boldsymbol{d} = \mathbf{0}$) and the ruptured state ($\boldsymbol{d} = \mathbf{1}$). In general, a crack in a material describes a strong discontinuity in a domain (Fig. 1, solid bold curve). To smoothen this discontinuity over the domain, the length scale parameter $\boldsymbol{l}$ is used, defined by the mesh size.

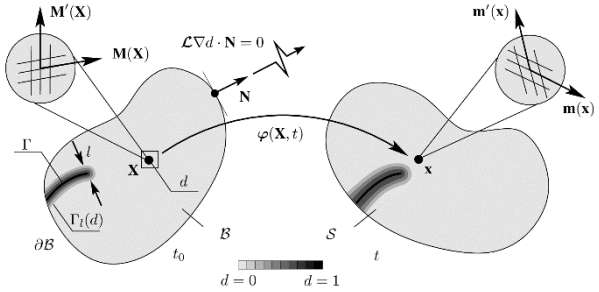


*Figure 1: Overview of an anisotropic continuum body with a sharp crack surface $\Gamma$, and a diffusive crack surface $\Gamma_l(d)$.*

The regularized anisotropic crack surface, which is based on the isotropic crack surface in [1], is described by

$$\Gamma_l(d) = \int_{\mathcal{B}} \frac{1}{2l}(d^2 + \nabla d \cdot \boldsymbol{\mathcal{L}} \nabla d)\,\mathrm{d}V \qquad (1)$$

where

$$\boldsymbol{\mathcal{L}} = l^2(\boldsymbol{I} + \omega_M \mathbf{M} \otimes \mathbf{M} + \omega_{M'} \mathbf{M}' \otimes \mathbf{M}') \qquad (2)$$

is defined as a second-order anisotropic structure tensor. With the implementation of the tensor $\boldsymbol{\mathcal{L}}$, the penalty term for the first fiber family $\boldsymbol{\omega_M}$ and for the second fiber family $\boldsymbol{\omega_{M'}}$ is included in the developed algorithm, see [2]. To solve the problem, the following considerations were used in their strong form, and further with a standard Galerkin method in their weak form:

$$J\,\mathrm{div}(J^{-1}\boldsymbol{\tau}) + \rho_0 \tilde{\boldsymbol{\gamma}} = \mathbf{0} \qquad (3)$$

$$\eta \dot{d} = (1-d)\boldsymbol{\mathcal{H}} - [d - \tfrac{1}{2}\mathrm{Div}(\boldsymbol{\mathcal{L}}\nabla d)] \qquad (4)$$

Equation (3) describes the balance of linear momentum in a continuum body for the kinematics of the mechanical problem, with the Kirchhoff stress tensor $\boldsymbol{\tau}$ and the body force $\tilde{\boldsymbol{\gamma}}$. Crack propagation is defined through Eq. (4), where the first term of the right hand side determines the crack driving force with the crack driving source term $\boldsymbol{\mathcal{H}}$**.** This term can be defined by the combination of the isotropic (ground matrix) and the anisotropic (fibers) contribution of the material, and is further defined with the respective effective free-energy function and the regularized critical energy release rate. The second term in (4) defines the geometrical resistance of the material.

## Results

To verify the model for an aortic dissection problem a multi-layer cylindrical domain with different sizes and shapes of an initial tear was used to simplify the aorta [2]. Two different cardiac cycles were applied to this domain, one physiological and one supra-physiological. Figure 2 illustrates the evolution of the crack phase-field.

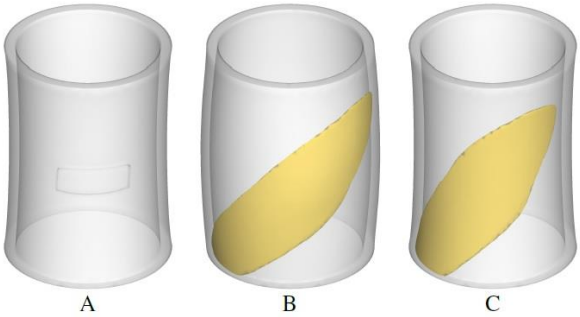


*Figure 2: Evolution of the crack phase-field; A: start of simulation, B: peak supra-physiological cycle, C: end of simulation.*

## Discussion

With the discussed model, it is possible to reproduce the same pattern of crack propagation as observed clinically.

## References

1. Miehe et al., Int. J. Numer. Meth. Eng., 83:1273-1311, 2010.
2. Gültekin et al., Biomech. Model. Mechanobiol., in press.