A 3D ELECTRO-MECHANICAL CONTINUUM MODEL FOR SKELETAL MUSCLE CONTRACTION

Belén Hernández-Gascón[1,2], Jorge Grasa[1,2], Begoña Calvo[1,2], José Félix Rodríguez[1,2]
[1] Applied Mechanics and Bioengineering, Aragón Institute of Engineering Research (I3A),
University of Zaragoza, Spain; [2] CIBBER-BBN, Centro de Investigación en Red en
Bioingeniería, Biomateriales y Nanomedicina, Spain

Introduction

Skeletal muscles are responsible for the body motion and posture support. Due to the cross-bridge cycling, muscle fibres are capable of generating force during isometric, concentric or eccentric contractions. This work proposes a 3D thermodynamically consistent electro-mechanical continuum model to simulate the skeletal muscle contraction.

Methods

Mathematical model. The deformation associated to muscle behaviour is modelled as two fictitious processes [Stålhand et *al*, 2011]: the first one is related to the relative movement between actin and myosin ($\mathbf{F_a}$), and the second one is associated to the elastic deformation of the cross bridges ($\mathbf{F_e}$), $\mathbf{F}=\mathbf{F_e}\cdot\mathbf{F_a}$. The anisotropy is described by unit vectors corresponding to the active, m_0, and passive responses, n_0. Let us assume that muscle fibres contract along the direction m_0 only and the existence of a strain energy function (SEF):

$$\Psi=\Psi_{vol}(J)+\bar{\Psi}_p(\bar{\mathbf{C}},\mathbf{N})+\bar{\Psi}_a(\bar{\mathbf{C}}_\mathbf{e},\bar{\lambda}_a,\mathbf{M}) \quad (1)$$

According to the Clausius-Planck inequality, the contraction velocity has to satisfy:

$$P_a-\frac{\partial\bar{\Psi}}{\partial\bar{\lambda}_a}+\left(2\bar{\mathbf{C}}_\mathbf{e}\frac{\partial\bar{\Psi}}{\partial\bar{\mathbf{C}}_\mathbf{e}}\bar{\mathbf{F}}_\mathbf{a}^{-T}\right):\frac{\partial\bar{\mathbf{F}}_\mathbf{a}}{\partial\bar{\lambda}_a}=C\dot{\bar{\lambda}}_a \quad (2)$$

where $C=C(\bar{\lambda}_a)\geq 0$.

FE model. A 3D FE model of the tibialis anterior (TA) rat muscle based on MR images is used to simulate isometric and concentric muscle contractions and to validate the proposed model considering previous experimental data, Figure 1. Specifically, we assumed:

$$\bar{\Psi}_a=P_0 f_1(\bar{\lambda}_a)f_2(f_r,t)\bar{\Psi}'_a(\bar{J}_a) \quad (3)$$

Results

Computational results after an isometric contraction fit the experimental results [Ramírez et *al*, 2010] and concentric contractions with different loads show that the contraction velocity decreases as the load increases, Figures 1 and 2.

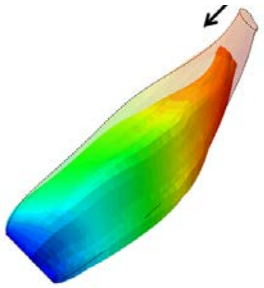

Figure 1: Superimposed view of the FE model before and after a concentric contraction.

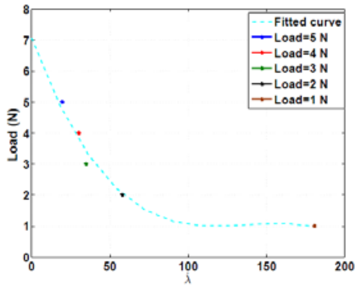

Figure 2: Force-velocity relationship obtained in concentric contractions with different loads.

Discussion

The proposed model is thermodynamically consistent and is capable of simulating the skeletal muscle contraction to obtain the evolution of active stresses and muscle length. Furthermore, the model does not depend on a velocity function, but the model is capable of reproducing such effect.

References

Ramírez *et al*, J Theor Biol, 267:546-553, 2010.
Stålhand *et al*, J Theor Biol, 268:120-130, 2011.