A DIFFERNTIAL HARDENING PLASTICITY MODEL FOR TRABECULAR BONE

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Introduction

Upon implantation of a press-fit orthopaedic implant, such as a femoral stem, the trabecular bone (TB) surrounding the implant is subjected to complex multiaxial loading and plastic deformation. Experimental work by Kelly et al. [2012a] has shown that the post-yield hardening behaviour of TB is dependent on the loading mode. In the current study a constitutive model based on the concept of differential hardening is developed whereby the post-yield behaviour of TB is a function of both the equivalent deviatoric ($\varepsilon^p{}_{dev}$) and volumetric plastic strain ($\varepsilon^p{}_{vol}$). Complimentary experimental testing on bovine TB specimens is used to calibrate and validate the material model.

Methods

Computational: A plasticity model is developed, featuring an anisotropic, pressure dependent yield surface, $f_y(\boldsymbol{\sigma})$. The evolution of the yield surface is described by a non-associated flow rule and a unique multiaxial hardening function, κ ($\varepsilon^p{}_{dev}$, $\varepsilon^p{}_{vol}$), which governs the post-yield stress depending on the degree of deviatoric (shape changing) to volumetric (volume changing) plastic strain in the TB:

$$\kappa\left(\varepsilon^p_{dev}, \varepsilon^p_{vol}\right) = \frac{\partial \kappa}{\partial \varepsilon^p_{dev}} \varepsilon^p_{dev} + \frac{\partial \kappa}{\partial \varepsilon^p_{vol}} \varepsilon^p_{vol} + \kappa^0 \quad (1)$$

$\partial\kappa/\partial\varepsilon^p$ is the hardening rate and the strain measures are calculated incrementally as follows:

$$d\varepsilon^p_{vol} \equiv d\varepsilon^p_{kk} = d\lambda \frac{\partial f_p(\boldsymbol{\sigma})}{\partial \sigma_{kk}} \quad (2)$$

$$d\varepsilon^p_{dev} \equiv \sqrt{\frac{2}{3} d\varepsilon^p_{ij}{}' d\varepsilon^p_{ij}{}'} \qquad d\varepsilon^p_{ij}{}' = d\lambda \left[\frac{\partial f_p(\boldsymbol{\sigma})}{\partial \sigma_{ij}} - \beta \alpha_{ij} \delta_{ij}\right] \text{(3a,b)}$$

where α and β are anisotropy constants and all other symbols and index notation assume their typical meanings in the context of multiaxial plasticity (e.g. Deshpande et al. [2000]). The hardening function is calibrated using experimental data representative of deviatoric and volumetric loading.

Experimental: Bovine proximal tibial TB specimens were subjected to three different loading modes: confined compression, unconfined uniaxial compression, and uniaxial torsion. It should be noted that the ratio of hydrostatic to deviatoric strain is different for each of these loading cases.

Results

Figure 1 shows a good match between the experimental results and computational predictions for each of the loading cases investigated. All computational curves were calculated using a single unique hardening function, demonstrating the ability of the constitutive model to capture multiaxial hardening of TB.

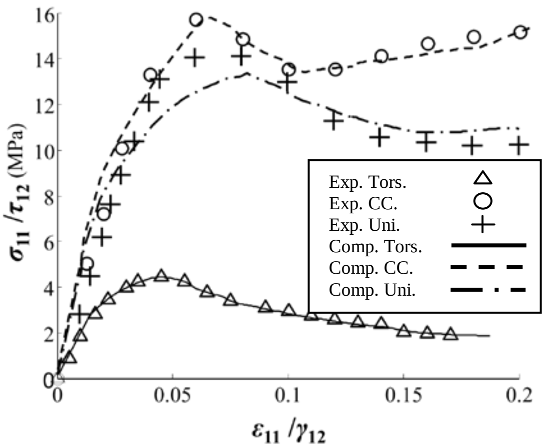

***Figure 1**. Nominal stress-strain curves for computational (Comp) and mean experimental (Exp) results of confined compression (CC), uniaxial compression (Uni) and torsional loading*

Discussion

The proposed plasticity model allows accurate predictions of post-yield behaviour in TB for a given loading path in $\varepsilon^p{}_{dev}$-$\varepsilon^p{}_{vol}$ space. A previous study of vertebral subsidence by Kelly et al. [2012b] demonstrated the inadequacy of commonly implemented plasticity laws in predicting TB deformation. The formulation presented in the current study provides, for the first time, accurate predictions of differential hardening in TB.

References

Deshpande *et al*, J Mech Phy Sol, 48:1253-83, 2000
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D.R.N. is funded by the Irish Research Council