NON-LINEAR VISCO-ELASTIC CONSTITUTIVE MODEL OF BONE TISSUE

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Introduction

In the paper a constitutive model for bovine cortical bone tissue is proposed. The model was derived from a postulated strain energy function W_e. In the presented approach it is assumed that bone is a visco-elastic and transversely isotropic material. Simulations with the implemented equation and remodelling phenomenon can be utilised in the process of an orthopaedic device design, e.g. intervertebral disc prosthesis design.

Method

The general constitutive equation for a visco-elastic material can be written in the form [Goh *et al.*,2004]:

$$S(\lambda,t)=\int_0^t g(t-s)\frac{\partial S^e(\lambda)}{\partial\lambda}\frac{\partial\lambda}{\partial s}\mathrm{d}s$$

where: S – second Piola-Kirchhof stress, λ – stretch ratio, g – time-dependent function, S^e – elastic part of second Piola-Kirchhoff stress. The function g is of the form:

$$g(t)=g_\infty+\sum_{i=1}^{n} g_i\cdot \mathrm{e}^{-\frac{t}{\tau_i}}$$

where: τ_i – relaxation times, g_i – characteristic amplitudes, $g_\infty=1-\sum_{i-1}^{n} g_i$. The stress S^e was derived from: [Holzapfel, 2000]:

$$S_{ij}^e=2\frac{\partial W_e}{\partial C_{ij}}$$

where: C_{ij} is right Cauchy tensor. The form of strain energy function is postulated to be:

$$W_e=c_1(I_1-3)^2+c_2\ln\left(1+c_3(I_4-1)^2\right)$$

The constants c_1, c_2, c_3 were identified on the basis of monotonic compression tests performed at three strain rates, i.e. $\dot{\lambda}=0.1$ min^{-1}, $\dot{\lambda}=0.5$ min^{-1} and $\dot{\lambda}=1.0$ min^{-1}; I_1 and I_4 are invariants of C_{ij}. The visco-elastic constants τ_i and g_i were identified on the basis of relaxation test.

Results

The curve fitting for the relaxation test is shown in Fig. 1.

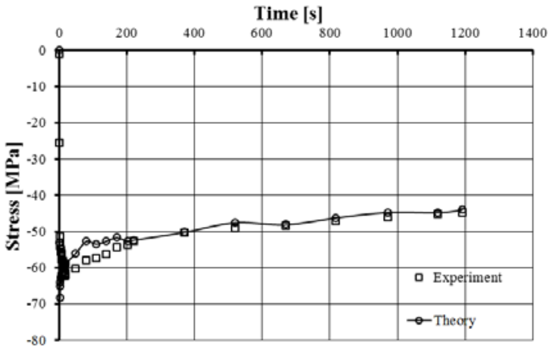

Figure 1: Relaxation curve fitting

The identified constants are listed in Table 1.

$c_1=70$ MPa	$g_1=0.199$	$\tau_1=0.1$ s
$c_2=58.9$ MPa	$g_2=0.306$	$\tau_2=2.34$ s
$c_3=49.08$	$g_3=0.0588$	$\tau_3=19.48$ s
	$g_4=0.0282$	$\tau_4=54.77$ s
	$g_5=0.246$	$\tau_5=1281.9$ s
	$g_6=0.0616$	$\tau_6=3666.7$ s

Table 1: Identified constants of bone

Discussion

The formulated constitutive equation can be implemented into a finite element system. Advanced FE analyses with the remodelling phenomenon taken into account allow one to simulate bone behaviour after, for instance, a new prosthesis implantation.

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References

Goh *et al.*, Mech Non-linear Mat, 8:255-268, 2004
Holzapfel, J. Wiley&Sons, Ltd, 2000.